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LETTER TO THE EDITOR

The four-dimensional XY spin glass

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Abstract. The nearest-neighbour XY spin glass on a hypercubic lattice in four dimensions is studied by Monte Carlo simulations. A finite-size scaling analysis of the data leads to a finite-temperature spin glass transition at $T_c = 0.95 \pm 0.15$. The critical exponents are estimated to be $\nu_{sg} = 0.70 \pm 0.10$ and $\eta_{sg} = -0.28 \pm 0.38$. The results imply that the lower critical dimensionality for the XY spin glass is less than four.

Recently there has been considerable interest in the behaviour of short-range vector spin glasses [1–8]. Although there is now convincing evidence [2–4] that XY spin glasses exhibit only a zero-temperature phase transition for $d = 2$ and 3, the location of the lower critical dimension, d_L , remains controversial. A zero-temperature study by Morris *et al* [2] suggests that $d_L = 4$. The validity of Nishimori and Ozeki's [5] attempt at a Mermin [9] type argument has been questioned [6] and it is claimed by Schwartz and Young [7] that all one can actually show is that $d_L \geq 2$.

A recent Migdal–Kadanoff renormalization-group analysis [8], on the other hand, claims that the XY spin glass orders at a finite temperature for $d = 4$ and, therefore, $d_L < 4$. The XY spin glass, unlike the corresponding gauge glass, possesses a 'reflection' symmetry. It has, therefore, been argued [10, 11] that the two models belong to different universality classes. Recent computer simulations [11] of the gauge glass in four dimensions clearly show a finite-temperature transition.

Furthermore, evidence for a finite-temperature transition in the four-dimensional Heisenberg spin glass with Gaussian nearest-neighbour interactions has been presented very recently by Coluzzi [12]. However, it is by no means clear that Heisenberg and XY spin glasses have the same lower critical dimension.

As far as we are aware, to date the XY spin glass has not been studied by simulations for $d = 4$. We attempt to fill this gap by presenting in this letter the first results of extensive Monte Carlo simulations of the four-dimensional XY spin glass with $\pm J$ nearest-neighbour interactions. Applying finite-size scaling theory [13], we shall find evidence for a finite-temperature glass transition.

The Hamiltonian for the model simulated is given by

$$H = - \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j = - \sum_{\langle i,j \rangle} J_{ij} \cos(\theta_i - \theta_j) \quad (1)$$

with $0 \leq \theta_i \leq 2\pi$ for all i . The planar spins, $\mathbf{S}_i (= (S_{i,x}, S_{i,y}))$, are situated on every site of a four-dimensional hypercubic lattice of size L^4 ($L = 2, 4$ and 6). The summation runs over nearest-neighbour pairs only and the interactions, J_{ij} , are independent random

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variables selected from a binary ± 1 distribution. As usual, the temperature is given in units of the nearest-neighbour interaction. We work with full periodic boundary conditions.

In the simulations we study the spin glass susceptibility, χ_{sg} , which is defined by

$$\begin{aligned}\chi_{\text{sg}} &= \frac{1}{N} \sum_{i,j} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle_T^2]_J \\ &= N q_{\text{sg}}^{(2)}\end{aligned}\quad (2)$$

where $\langle \dots \rangle_T$ indicates a thermal average, $[\dots]_J$ denotes an average over the disorder, $N = L^4$ and

$$q_{\text{sg}}^{(2)} = \sum_{\mu,\nu} [\langle q_{\mu\nu}^2 \rangle_T]_J. \quad (3)$$

In equation (3) the tensor variable, $q_{\mu\nu}$, is defined in terms of the overlap between two replicas 1 and 2,

$$q_{\mu\nu} = \frac{1}{N} \sum_i S_{i,\mu}^1 S_{i,\nu}^2 \quad (\mu, \nu = x, y). \quad (4)$$

Higher-order correlations such as $q_{\text{sg}}^{(4)}$ can also be written in terms of $q_{\mu\nu}$, namely

$$q_{\text{sg}}^{(4)} = \sum_{\mu,\nu,\alpha,\beta} [\langle q_{\mu\nu}^2 q_{\alpha\beta}^2 \rangle_T]_J. \quad (5)$$

Rather than use $q_{\text{sg}}^{(2)}$ and $q_{\text{sg}}^{(4)}$, it is far more convenient to work with the dimensionless Binder parameter [13] defined by

$$g_{\text{sg}} = 3 - 2 \frac{q_{\text{sg}}^{(4)}}{(q_{\text{sg}}^{(2)})^2}. \quad (6)$$

According to finite-size scaling theory [13], near T_c we expect the Binder parameter to scale as

$$g_{\text{sg}}(L, T) = \bar{g}_{\text{sg}}(L^{1/\nu_{\text{sg}}}(T - T_c)) \quad (7)$$

where ν_{sg} is the correlation length exponent and \bar{g}_{sg} is a scaling function satisfying

$$\bar{g}_{\text{sg}}(L^{1/\nu_{\text{sg}}}(T - T_c)) = \begin{cases} 0 & \text{for } T > T_c, L \rightarrow \infty \\ 1 & \text{for } T < T_c, L \rightarrow \infty \end{cases} \quad (8)$$

provided that the ground state is non-degenerate. Whereas for a finite-temperature spin glass transition plots of $g_{\text{sg}}(L, T)$ against T for different L should intersect at T_c , for a transition at zero temperature we expect the curves to meet each other at $T = 0$ only.

The scaling form for the spin glass susceptibility is given by

$$\chi_{\text{sg}}(L, T) = L^{2-\eta_{\text{sg}}} \bar{\chi}_{\text{sg}}(L^{1/\nu_{\text{sg}}}(T - T_c)) \quad (9)$$

where the exponent η_{sg} describes the power-law decay of correlations at the transition temperature and $\bar{\chi}_{\text{sg}}$ is now a scaling function. It follows from equation (9) that

$$\chi_{\text{sg}}(L, T_c) \sim L^{2-\eta_{\text{sg}}}. \quad (10)$$

As the coordination number for our model is $Z = 8$, the mean-field values of T_c and the exponents mentioned above are

$$T_c^{\text{mf}} \approx \sqrt{Z}/2 = \sqrt{2} \quad \nu^{\text{mf}} = \frac{1}{2} \quad \eta^{\text{mf}} = 0. \quad (11)$$

We now describe our Monte Carlo simulations and discuss the results. During the simulations, which were performed using the conventional Metropolis [14] technique on a Cray Y-MP8 and a J90, we actually work with discrete spins. For technical reasons, the spins were discretized to occupy 256 equally spaced orientations in the plane. Furthermore, a variant of multispin coding [15] was used to store seven (discrete) spins in one word and the lattice was composed of two inter-penetrating sublattices. As a consequence, each update of the lattice allows us to update 14 separate samples (or, alternatively, seven pairs of samples) at the same time.

We follow Bhatt and Young [13] and compare the spin glass correlations obtained from two independent replicas with the same set of bonds with those from a single replica at two different times. Equilibrium is assumed only if the values agree within the statistical error.

The number of Monte Carlo steps, $\tau_0(L, T)$, required to achieve equilibrium depends on both the system size L and the temperature T . $\tau_0(L, T)$ sets upper and lower limits on the values of L and T , respectively, that can be studied. In our simulations we found that $\tau_0(2, 0.4) \approx 1000$, $\tau_0(4, 0.7) \approx 5000$ and $\tau_0(6, 0.75) \approx 13000$ sweeps; equilibration problems prevented us from going to lower temperatures. The number of bond configurations we generated to average over the disorder also varied with L . Typically, we considered 7000 ($L = 2$), 250 ~ 2300 ($L = 4$), 100 ~ 500 ($L = 6$) pairs of samples for each temperature considered. (However, note that for the lowest two temperatures ($T = 0.775$ and 0.75) for $L = 6$ we averaged over only 56 pairs of samples in each case.) In total, the simulations presented in this work took approximately 600 h of CPU time on the two supercomputers mentioned above.

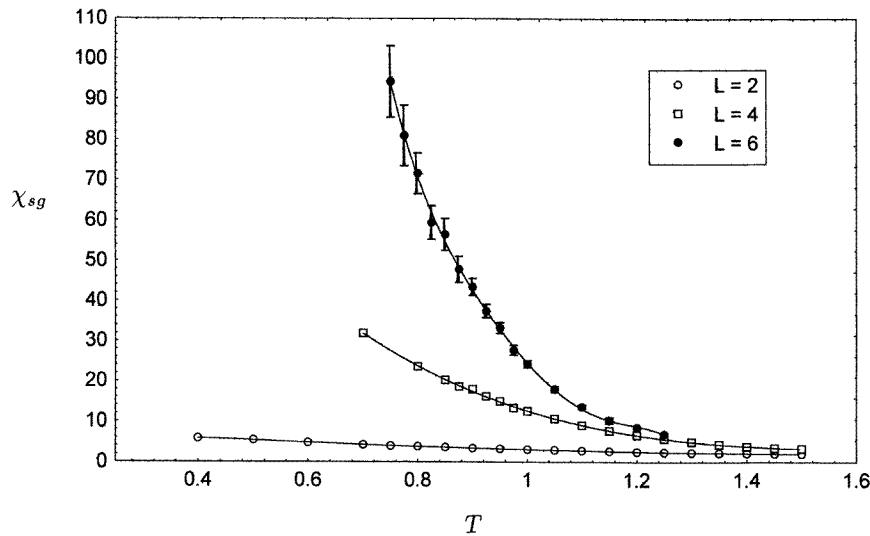


Figure 1. A plot of the spin glass susceptibility, χ_{sg} , against the temperature for $L = 2, 4$ and 6 . The lines are just guides to the eye.

In figure 1 we plot χ_{sg} against T for the three different values of L considered. The statistical error bars were evaluated from the sample-to-sample fluctuations and are only displayed in those cases where they exceed the size of the points.

The Binder parameter is plotted against the temperature in figure 2. Although the curves clearly intersect at a finite temperature, the point of intersection is not unique. This

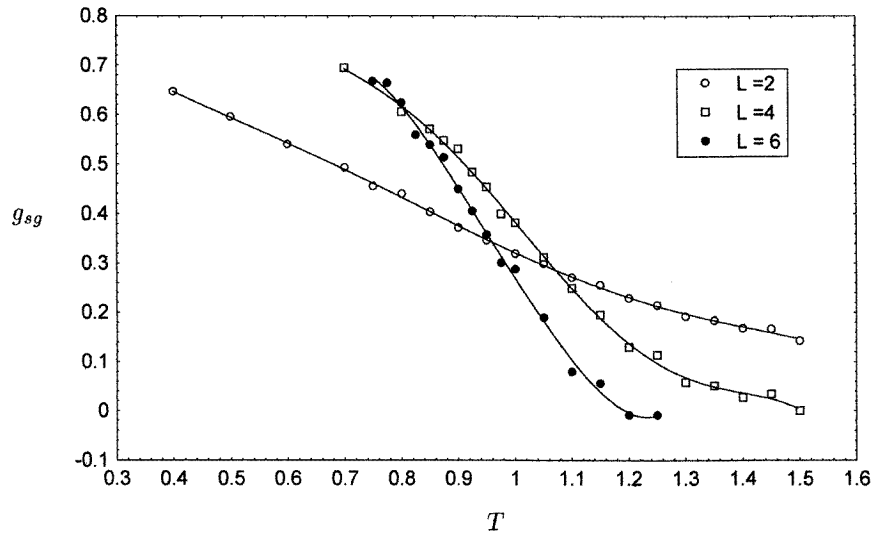


Figure 2. A plot of the Binder parameter defined in equation (6) against the temperature for $L = 2, 4$ and 6 . The lines are just guides to the eye.

is probably due to corrections to finite-size scaling and the statistical error in g_{sg} . As we have data for three values of L , we obtain three intersection temperatures, $T_c^{L_1, L_2}$, where $L_1, L_2 = 2, 4, 6$ and $L_1 \neq L_2$. For the data presented in figure 2 we note that there is a small downward shift in the value of $T_c^{L_1, L_2}$ for increasing L_1 and L_2 . Clearly, to establish whether the shift is significant or not, it is highly desirable to obtain additional data for the Binder parameter for larger lattices and lower temperatures. However, we note that there is some evidence of a finite-temperature transition as the curves clearly splay out below the intersection point. From figure 2 we estimate the spin glass transition temperature to be $T_c = 0.95 \pm 0.15$. Our value of $T_c (\approx 0.7 T_c^{\text{mf}})$ agrees well with the value of $T_c \approx 0.9$ obtained recently by Nobre *et al* [8]. It is also surprisingly close to the transition temperature found by Reger and Young [11] for the four-dimensional *gauge* glass.

Having obtained an estimate for T_c , a log-log plot of $\chi_{\text{sg}}(L, T_c)$ against L is expected from equation (10) to have a slope of $2 - \eta_{\text{sg}}$. Our results are consistent with this but the uncertainty in T_c leads to a large error in η_{sg} and we estimate that $2 - \eta_{\text{sg}} = 2.28 \pm 0.38$.

To fix the second independent exponent, ν_{sg} , we display in figure 3 a scaling plot of $\chi_{\text{sg}}(L, T)/L^{2.28}$ against $(L^{1/\nu_{\text{sg}}}(T - 0.95))$. To see how sensitive the scaling plot is to the value of ν_{sg} , we have tried a range of values. As a result we estimate $\nu_{\text{sg}} = 0.70 \pm 0.10$. As can be seen from figure 3, the data scale reasonably well for $\nu_{\text{sg}} = 0.70$. Once again, our value for the correlation exponent is remarkably close to the one found for the gauge glass in four dimensions [11].

Finally, in figure 4 we show a scaling plot of the data for the Binder parameter for the *same* values of T_c and ν_{sg} as the ones used in the plot for figure 3. We see that the data for g_{sg} do not scale as well as those for χ_{sg} . The quality of the data collapse does not improve for other possible values of T_c and ν_{sg} . (A correction to scaling as suggested by Bokil and Young [16] also fails to make any significant difference to the plot.) A similar behaviour was found by Kawamura [4] in the three-dimensional *XY* spin glass.

It has been assumed that the *XY* spin glass and the gauge glass belong to different

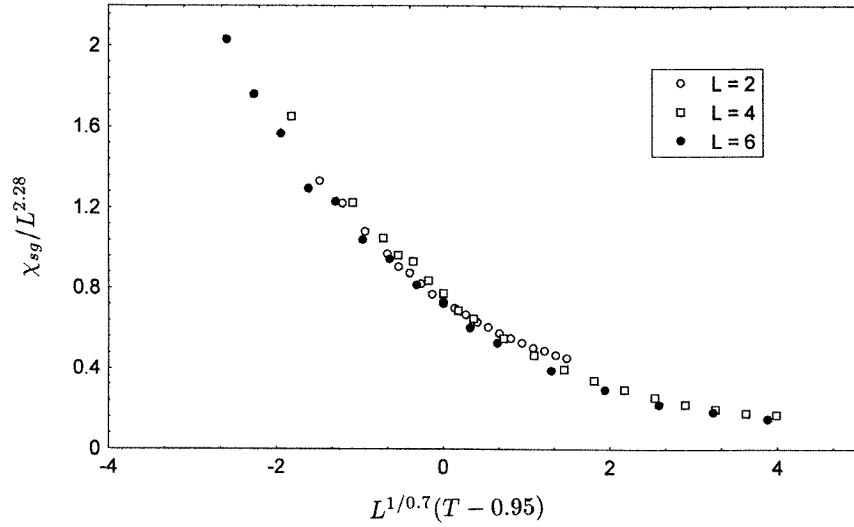


Figure 3. A scaling plot of $\chi_{sg}/L^{2-\eta_{sg}}$ against $L^{1/\nu_{sg}}(T - T_c)$ with $\eta_{sg} = -0.28$, $\nu_{sg} = 0.70$ and $T_c = 0.95$. See equation (9) in the text.

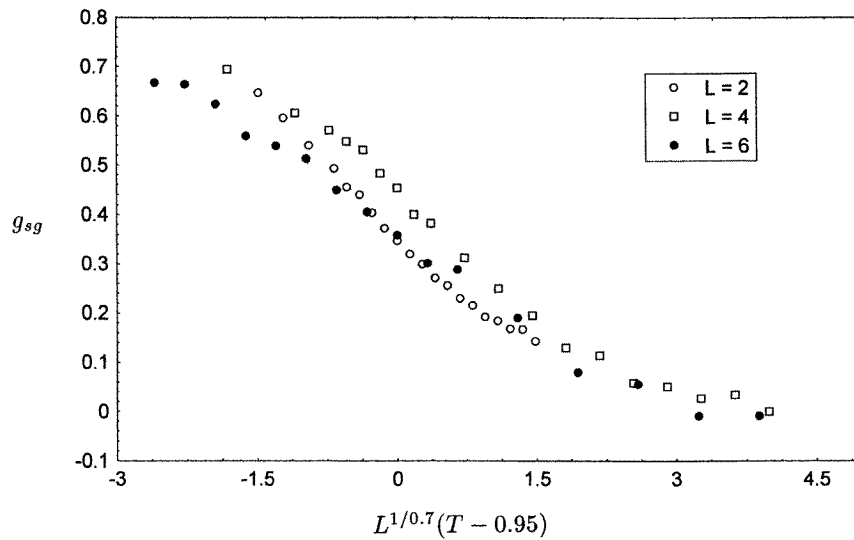


Figure 4. A scaling plot of the Binder parameter g_{sg} against $L^{1/\nu_{sg}}(T - T_c)$ for $\nu_{sg} = 0.70$ and $T_c = 0.95$.

universality classes as the latter does not share the reflection symmetry of the former. As noted, our results are remarkable in their similarity to those found earlier by Reger and Young [11] for the vortex glass in four dimensions. It is felt that this unexpected feature requires further investigation.

To conclude, we have presented numerical evidence that the XY spin glass has a finite-temperature glass transition in four dimensions. We have estimated the critical temperature

and the critical exponents. Further work is required to confirm the transition temperature and obtain more accurate values for the exponents. Our results are in agreement with the analytic approximation carried out by Nobre *et al* [8]. They are also surprisingly similar to those found earlier by Reger and Young [11] for the four-dimensional gauge glass and imply that $d_L < 4$ for the *XY* spin glass.

Work is underway to investigate the chiral glass [4, 16] behaviour of the model using the vortex representation [16].

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References

- [1] Banavar J R and Cieplak M 1982 *Phys. Rev. Lett.* **48** 832
McMillan W L 1985 *Phys. Rev. B* **31** 342
Olive J A, Young A P and Sherrington D 1986 *Phys. Rev. B* **34** 6341
Matsubara F and Iyota and Inawashiro 1991 *Phys. Rev. Lett.* **67** 1458
Kawamura H 1992 *Phys. Rev. Lett.* **68** 3785; 1995 *J. Phys. Soc. Japan* **64** 26
Coluzzi B 1995 *J. Phys. A: Math. Gen.* **28** 747
- [2] Morris B W, Colborne S G, Moore M A, Bray A J and Canisius 1986 *J. Phys. C: Solid State Phys.* **19** 1157
- [3] Jain S and Young A P 1986 *J. Phys. C: Solid State Phys.* **19** 3913
Ray P and Moore M A 1992 *Phys. Rev. B* **45** 5361
- [4] Kawamura H 1995 *Phys. Rev. B* **51** 12 398
Kawamura H and Tanemura M 1985 *J. Phys. Soc. Japan* **54** 4479; 1986 *J. Phys. Soc. Japan* **55** 1802
- [5] Nishimori H and Ozeki Y 1990 *J. Phys. Soc. Japan* **59** 295
Ozeki Y and Nishimori H 1992 *Phys. Rev. B* **46** 2879
- [6] O'Neill J A and Moore M A 1990 *J. Phys. Soc. Japan* **59** 289
- [7] Schwartz M and Young A P 1991 *Europhys. Lett.* **15** 209
- [8] Nobre F D, Mariz A M and Sousa E S 1993 *Physica A* **196** 505
- [9] Mermin N D 1967 *J. Math. Phys.* **8** 1061
- [10] Reger J D, Tokuyasu T A, Young A P and Fisher M P A 1991 *Phys. Rev.* **44** 7147
Fisher M P A, Tokuyasu T A and Young A P 1991 *Phys. Rev. Lett.* **66** 2931
Huse D A and Seung H S 1990 *Phys. Rev. B* **42** 1059
- [11] Reger J D and Young A P 1993 *J. Phys. A: Math. Gen.* **26** L1067
- [12] Coluzzi B 1995 *J. Phys. A: Math. Gen.* **28** 747
- [13] Bhatt R N and Young A P 1988 *Phys. Rev. B* **37** 5606
- [14] Metropolis N, Rosenbluth A W, Rosenbluth M N, Teller A H and Teller E 1953 *J. Chem. Phys.* **21** 1087
- [15] Bhanot E, Duke D and Salvador R 1986 *J. Stat. Phys.* **44** 85; 1988 *Comput. Phys. Commun.* **49** 465
- [16] Bokil H S and Young A P 1996 *J. Phys. A: Math. Gen.* **29** L89